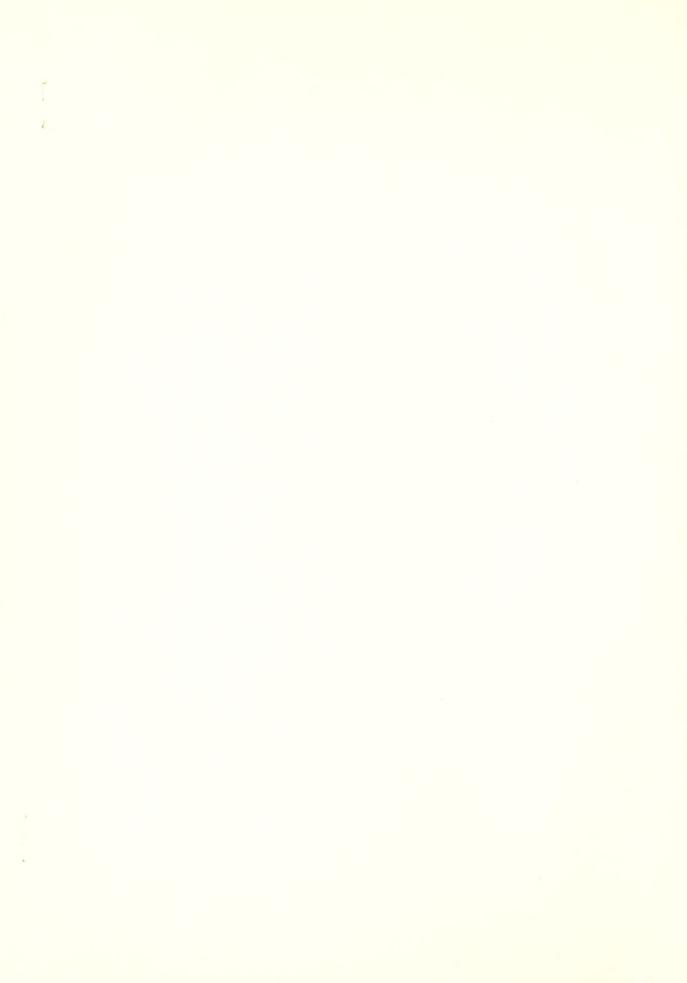
OPTIMUM AIRCRAFT WEAPONS SELECTION AND SEQUENTIAL FIRING STRATEGY FOR TARGETS OF OPPORTUNITY

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OPTIMUM AIRCRAFT WEAPONS SELECTION AND SEQUENTIAL FIRING STRATEGY FOR TARGETS OF OPPORTUNITY

bу

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A pilot engaged in a strike mission against targets of opportunity is continually confronted with the decision as to how many and which of his available weapons should be expended against a variety of targets which are encountered. Associated with this decision problem is the problem of determining an optimal load for such a mission subject to the constraints of available payload and number of weapons stations on the aircraft. Certain assumptions are made concerning the distribution of targets within the target area which lead to a dynamic programming formulation of the decision problem. This yields a system of ordinary differential equations which are solvable recursively. In addition to the dynamic programming model, a sub-optimum determination of a "best load" is discussed. Although not as complete or precise as the dynamic programming method, this formulation is more readily adaptable to squadron level decision making.

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1. INTRODUCTION

The problem of selecting weapons for an aircraft strike mission and using those weapons effectively has been documented quite extensively. However, present publications are devoted almost exclusively to the situation where the target is assumed known prior to the mission and the aircraft can be loaded for that target, or where several different aircraft can be loaded differently to cover an unknown spectrum of targets. 1 These approaches are inadequate for determining weapons requirements against targets of opportunity. A pilot engaged in such a mission is confronted with the decision as to which and how many of his available weapons to expend when any target is encountered. When various types of targets might be encountered, each with a different strategic or tactical value, he wishes to maximize the value of the targets he can expect to destroy. Associated with this is the problem involving the choice of weapons that should be initially loaded onto the aircraft for such a mission. Here the problem is constrained by maximum payload weapons and weapons stations availability. It is desired to obtain, within these constraints, (1) the optimum feasible weapons mix to be loaded for use against an uncertain spectrum of targets, and (2) a doctrine for the expenditure of these weapons when targets are encountered.

The model developed herein attacks these problems in reverse order. First, an optimum firing doctrine is generated for all feasible loads. Then that load which yields the maximum expected return,

NAVWEPS Report 8664, "Preferred Aircraft Loadings Based on the Predicted Combat Effectiveness of Conventional Air-to-Surface Weapons," NOTS TP 3710, 1965, (SECRET).

when expended in accordance with the optimum doctrine, becomes the optimal load.

ASSUMPTIONS IN THE MODELS

Certain assumptions, necessary to the model development are made concerning target distributions within the target area and tactics of the strike aircraft.

It is assumed that there are a finite number of different target types, each randomly and independently distributed within the target area with known density. For example, from photo-reconnaissance or other sources it might be known that the density of trucks within the target area is x per square mile. The number of targets of each type is assumed to be large enough so that the probability of future encounters is unchanged by consideration of the number already met. Further, these targets are assumed to be of a unitary nature, characterized by the existence of single shot kill probabilities, as distinguished from area targets characterized by probabilities of destroying a certain percentage of the area or value of the target.

The assumption is made that a ratio scale of target values can be specified. For example, with the least valuable target as a base with value V, the achievement of a kill on a target type i is worth 7V, target type j is worth 8V, etc.

For defensive reasons the strike aircraft is restricted to a single attack on any target which might be encountered. The pilot may not drop a bomb on the target, assess damage, and then make another attack if a kill has not been achieved. He may however, expend any number of weapons on this single pass provided they are all

not drop one 500 lb. bomb and one 1,000 lb. bomb. This restriction is not necessary to the development of the models, but rather recognizes that differences in aim points between different weapons make this mixed drop undesirable.

METHOD OF SOLUTION

Using data on weapons weights, constraints of aircraft payload capacity, and the number of available weapons stations on the plane, all feasible loadings can be enumerated. Initial loads are then considered which are not subsets of some other feasible load. That is, if five weapons of type one and three weapons of type two constitute a feasible load, then four weapons of type one and three of type two is also feasible but a subset of the former. The kill potential of some subset of a feasible load is clearly less than that of the original load and is therefore not considered in the determination of an optimal initial loading.

A functional equation for the expected value of targets killed is then set up as a dynamic programming process yielding a value for kill potential for each of the feasible loads being considered. The maximum kill potential from amongst these loadings determines the optimal load for the strike aircraft. Additionally in the solution of this dynamic programming process, the optimal firing strategy for the pilot is obtained for any initial load. The optimized sequential decision process yields a policy which determines as a function of the operating time and weapons remaining, the best weapons to be expended in an encounter with any of the various target types.

Dynamic programming provides a complete and very precise means of determining the optimum load for a "targets of opportunity" type mission. Except in the very simplest cases however, the problem is quite complex and will require a computer for solution. In the majority of operational situations such equipment will not be available. Thus an approximate method, suitable for hand calculation, is needed for determining the initial load.

In order to achieve simplicity, this "hand model" must sacrifice detail and thus completeness. In this respect it is a sub-optimal method. The simplified method is best suited for comparing a few, rather than all, feasible loads for use against a selected target list. By choosing the best weapon for each target type and comparing the expected returns for various mixes of these weapons, a "best load" can be generated.

2. GENERAL DISCUSSION AND DEVELOPMENT OF THE MODELS

The random distribution of targets leads to an exponential interencounter time distribution for the strike aircraft searching the target area. (All targets which are within the visibility range of the aircraft's track are assumed sighted. Alternatively the sightings could be reduced by some condation factor which takes into account the difficulty associated with perceiving a given target type. This however would not alter the analysis which follows.) The mean rate of sightings (the reciprocal of the mean of the exponential distribution of inter-encounter times) is denoted by r.

DYNAMIC PROGRAMMING MODEL

The development of the dynamic programming formulation of the problem is best seen by first considering the simpler cases. To begin with, a model has previously been developed for the allocation of weapons over a limited time interval. This can be utilized for generating the optimal firing strategy in the simplest case of one target type-one weapon type. The optimal loading problem is not pertinent; for a single weapon type the best load is clearly the maximum number that can be carried on the aircraft. First define:

l Appendix I

Bram, J., "Allocation of Weapons to Targets with Exponential Arrival Times in a Limited Time Interval," OEG Interim Research Memorandum 32, 1963.

An initial loading of N weapons and a time T allowable for search is assumed, and $f_n(t)$ will be calculated for $0 \le n \le N$ and $0 \le t \le T$. Define:

p = probability that j weapons expended against a target
j will achieve a kill.

Clearly $f_n(t)$ is dependent on the p 's and on r, the encounter rate.

By Bellman's "principle of optimality," for $0 \le t_1 \le t$, the optimal policy in the interval (0, t), given n weapons at time t = 0, must also be optimal in the subinterval (t_1, t) given $n - n_1$ weapons at time $t = t_1$, where n_1 weapons were expended in the first subinterval $(0, t_1)$.

Let $t = \Delta t \ll T$. By definition of the exponential arrival process, the probability of an encounter in a small interval of time Δt is proportional to Δt with constant of proportionality r, the encounter rate. Thus the probability of one encounter in the time interval $(0, \Delta t)$ is $r\Delta t$ and no encounters occur with probability $1 - r\Delta t$.

Breaking the time interval (0, t) into the two sub-intervals (0, Δt) and (Δt , t) yields:

 $(1 - r\Delta t)$ = the probability that no encounters will occur in the first sub-interval.

p = the expected return in the first sub-interval
 having expended j weapons.

Bellman, R., <u>Dynamic Programming</u>, (Princeton, New Jersey: Princeton University Press, 1957), p. 83.

f (t - Δt) = the expected return in the second sub-interval having expended no weapons in the first and following an optimal strategy in (Δt , t).

f (t - Δ t) = expected return in the second time interval having n-j expended j of the n weapons in the first subinterval and behaving optimally in (Δ t, t).

The expected return over the whole interval (0, t) is then:

$$(1 - r\Delta t) f_n(t - \Delta t) + r\Delta t (p_j + f_{n-j} (t - \Delta t))$$

To optimize over the period (0, t), j must be chosen such that this expression is a maximum. Therefore

(1)
$$f_n(t) = (1 - r\Delta t) f_n(t - \Delta t) + r\Delta t \max_{1 \le j \le n} (p + f_{n-j}(t - \Delta t))$$

The maximization here is over the range $1 \le j \le n$ rather than $0 \le j \le n$ since it clearly would never pay to let a target go by without firing any weapons, saving them all for future encounters which are not certain to occur.

Rearranging (1), dividing by Δt and taking the limit as Δt approaches zero yields:

(2)
$$f'_n(t) = -rf_n(t) + r \max_{1 \le j \le n} (p + f_{n-j}(t))$$

From (2), $f_n(t)$ can be calculated for all n and t recursively as follows: By definition, $f_n(t) = 0$ for all t. (The expected return is zero over any period of time if there are no weapons with which to operate.) Also $f_n(0) = 0$ for all n. (The expected return is zero if there is no time in which to operate.) Solving (2) for

n = 1 yields:

$$f_1(t) = p_1 (1 - e^{-rt})$$

Note that this result was to be expected. Exponential inter-encounter times lead to a Poisson counting process for the number of encounters within a time interval. The term $(1 - e^{-rt})$, is simply the probability of at least one encounter in the time interval (0, t), and p_1 is the probability that, given an encounter, a kill will be achieved with the expenditure of the single weapon.

Utilizing $f_1(t)$ equation (2) can now be solved for n = 2.

$$f'(t) = -rf_2(t) + r max (p_1(2 - e^{-rt}), p_2)$$

Since
$$p_2 > p_1(2 - e^{-rt})$$
 when either $t < \frac{1}{r} \ln \frac{p_1}{2p_1 - p_2}$ or if $p_2 > 2p_1$,

the solution to the differential equation is:

$$f_2(t) = p_2 (1 - e^{-rt})$$
 for $t \le \frac{1}{r} \ln \frac{p_1}{2p_1 - p_2}$ or if $p_2 \ge 2p_1$

$$f_2(t) = 2p_1 - e^{-rt} (rp_1t + p_1 + p_2 - p_1 ln \frac{p_1}{2p_1-p_2})$$

for
$$t \ge \frac{1}{r} \ln \frac{p_1}{2p_1 - p_2}$$
 and $p_2 < 2p_1$

The significance in this solution lies in the "cross-over" point in time. If the search time remaining is less than this value of t, the optimal return is achieved by dropping both bombs on the next target which is encountered. If greater, only one bomb is dropped on the target, the other being saved for possible future encounters.

Having found $f_2(t)$, $f_3(t)$ can now be calculated. The differential equation however becomes unduly complicated for n > 2. A numerical solution may be obtained for the problem though by recourse to equation (1), the difference equation, with which $f_n(t)$ can be calculated for any n to any desired degree of accuracy by taking Δt small enough.

With the possibility of encounters with targets of different types, the value of a target which is met must be considered when the decision is made as to how many weapons should be expended.

The methods used in developing the functional equation are analagous to those for the simpler model with the addition of considerations of target value and different kill probabilities. Define:

m = the number of different target types.

r = average encounter rate with type i targets.

v = value of target type i.

f_n(t) = expected value of targets killed in the interval (0, t)
 given n bombs at time t = 0 and optimal utilization of
 these weapons.

Then:

(3)
$$f_n(t) = (1 - \sum_{i=1}^{m} r_i \Delta t) f_n(t - \Delta t) + \sum_{i=1}^{m} r_i \Delta t \max_{0 \le j \le n} (v_i p_{ij} + f_{n-j}(t - \Delta t))$$

Note that in this case the maximization is over the range $0 \le j \le n$ vice $1 \le j \le n$. Here the expected return might be greater if no weapons are dropped on targets of little value, expecting at some later time in the mission to meet something of greater worth.

The differential equation is obtained by subtraction of $f_n(t)$ from both sides of the above equation, division by Δt and taking the limit as Δt approaches zero.

$$f'(t) = -\sum_{i=1}^{m} r_i f_n(t) + \sum_{i=1}^{m} r_i \max_{0 \le j \le n} (v_i p_{ij} + f_{n-j}(t))$$

For n = 1 and m = 2 the solution to this equation can be shown to be:

$$f_1(t) = \frac{(r_1^p_1^{v_1} + r_2^p_2^{v_2})}{(r_1 + r_2)} (1 - e^{-(r_1 + r_2)t}) \text{ for } p_2^{v_2} > p_1^{v_1}$$

and
$$0 \le t \le \frac{1}{r_1 + r_2} = \ln \frac{(r_1 p_1 v_1 + r_2 p_2 v_2)}{r_2 (p_2 v_2 - p_1 v_1)}$$

or if $p_1 v_1 > p_2 v_2$ then for

$$0 \le t \le \frac{1}{r_1 + r_2} = \ln \frac{(r_1^p_1^{v_1} + r_2^p_2^{v_2})}{r_1(p_1^{v_1} - p_2^{v_2})}$$

or if $p_1v_1 = p_2v_2$ then for all t.

$$f_1(t) = p_1 v_1 + (p_2 v_2 - p_1 v_1) \left[\frac{r_1 p_1 v_1 + r_2 p_2 v_2}{r_1 (p_1 v_1 - p_2 v_2)} \right] \frac{r_1}{r_1 + r_2} e^{-r_1 t}$$

for
$$p_1 v_1 > p_2 v_2$$
 and $t \ge \frac{1}{r_1 + r_2} \ln \frac{r_1 p_1 v_1 + r_2 p_2 v_2}{r_1 (p_1 v_1 - p_2 v_2)}$

$$f_1(t) = p_2 v_2 + (p_1 v_1 - p_2 v_2) \left[\frac{r_1 p_1 v_1 + r_2 p_2 v_2}{r_2 (p_2 v_2 - p_1 v_1)} \right] \frac{r_1}{r_1 + r_2} e^{-r_2 t}$$

for
$$p_2 v_2 > p_1 v_1$$
 and $t \ge \frac{1}{r_1 + r_2} \ln \frac{r_1 p_1 v_1 + r_2 p_2 v_2}{r_2 (p_2 v_2 - p_1 v_1)}$

Thus for the simple case of two target types and a single weapon, the expected returns can be calculated analytically. In addition, the optimal strategy is found, here consisting merely of a cross-over time, the time before which it does not pay to fire the weapon at the target with the lowest p v. For n > 1 however, it can readily be seen that i i solutions to the differential equation are not so easily obtained.

Again though, through recourse to the difference equation, (3), numerical solutions can be obtained for any n to any desired degree of accuracy.

Further generalization of the model entails a relaxation of the restriction to a single weapon type. The functional equation is similar to those for the simpler models with some minor changes in the definitions of the terms. Define:

L = the number of different weapon types.

f (t) = expected value of targets killed in the interval ${}^{n}_{1}{}^{n}_{2}{}^{n}_{3}{}^{\cdots}{}^{n}_{L}$ (0, t), given n weapons of type i (for i=1,...,L) at t = 0 and optimal utilization of these weapons.

The difference equation can be written:

$$f_{n_{1} n_{2} n_{3} ... n_{L}}$$
 (t) = $(1 - \sum_{i=1}^{m} r_{i} \Delta t)$ $f_{n_{1} n_{2} n_{3} ... n_{L}}$ (t - Δt)

$$+\sum_{i=1}^{m} r_{i} \Delta t \max_{1 \leq j \leq L} \max_{0 \leq k \leq n} (v_{i} p_{j} + f_{n, n, \ldots, n-k, \ldots, n} (t - \Delta t))$$

The differential equation can be written as before, but the solutions are obtainable only in the most trivial cases.

A FORTRAN program has been written however from which numerical solutions to the difference equation can be calculated. With inputs of the aircraft load carrying capabilities, the mission time, and the parameters of the model (m, r_i , v_i , L, p_{ijk} , and Δt) the program selects values for the n which are consistent with the limitations of the aircraft and maximize f. Additionally, another FORTRAN program lists the optimal strategy to be employed in the decisions relating to the expenditure of these weapons.

A SUB-OPTIMAL DETERMINATION OF LOAD

In the majority of practical cases hand calculation will be the only means available for determining loading requirements. In such cases a sub-optimal method can be used at the squadron level for obtaining the initial weapons load for a mission. Define:

T = search time (i.e., mission time allotted to searching for and attacking targets).

p (T) = probability that target type i will be met n times during the mission.

$$\Pi_{i}(T) = \sum_{n=1}^{\infty} p_{ni}(T) = \text{probability that target type i will be met at least once during the mission.}$$

v = value of target type i.

m = number of target types.

 $b_{js} = number of sticks^{1} of s weapons of type j in the load.$

L = number of weapon types available.

¹In this development the terms stick and salvo are used equivalently. The choice of a stick or a salvo, in the load determination, will depend upon which gives the higher kill probability.

k = probability that a stick of s type w weapons will kill
iws target type i.

Note that in this development individual bombs are considered as a stick of one; i.e., two 250 lb. bombs might be considered as two sticks of one type j or one stick of two type j weapons.

If, a priori, only the more highly valued targets and those individual weapons with the highest kill probabilities against these targets are considered, the assumption can be made that a target sighted will be dropped on, as long as the appropriate weapon for that target is available. (Note that this assumption is used only for determining the load. In the actual performance of the mission there is the possibility that, if that weapon most effective against a target is not available, a less effective weapon will be dropped.) The problem then becomes one of maximizing for a given mission time, the expected value of targets killed, E, for those loads being considered. E may be determined as follows:

$$E = \sum_{i=1}^{m} f_{ijs} k_{ijs} v_{i}$$

where f_{ijs} is the expected number of type i targets on which weapon type j can be expended, and j indicates that weapon type for which $k_{ij1} \ge k_{iw1}$ for all $w = 1, 2, \cdots, L$.

If the number of type j weapons available were limitless, figswould simply be equal to $\frac{1}{n} = \sum_{n=0}^{\infty} np_{ni}(T)$. However, since a given load is limited to b sticks of type j weapon, no more than b can be

dropped regardless of how many type i targets are sighted. Thus

The decision as to which targets and weapons will be considered will depend highly upon the judgement of the individual responsible for determining the aircraft load.

f_{ijs} is given by

(4)
$$f_{ijs} = \sum_{n=0}^{b_{js}-1} np_{ni}(T) + \sum_{n=b_{js}} b_{js}p_{ni}(T)$$

Equation (1) can be reduced to a more easily calculable form.

Since
$$b_{js}^{-1}$$
 b_{js}^{-1} ∞ $\sum_{n=0}^{\infty} np_{ni}(T) = \sum_{n=1}^{\infty} np_{ni}(T)$ and $\sum_{n=b_{js}}^{\infty} p_{ni}(T) = \prod_{i} \sum_{n=1}^{\infty} p_{ni}(T)$,

equation (4) can be written as

$$f_{ijs} = \sum_{n=1}^{b_{js}-1} np_{ni}(T) + b_{js} \frac{n}{i} - b_{js} \sum_{n=1}^{b_{js}-1} p_{ni}(T)$$

$$= b \prod_{js i}^{n} - \sum_{n=1}^{b_{js}-1} (b_{js}-n) p_{ni}(T)$$

but $\Pi_i = 1 - p_{0i}(T)$ and consequently $b \Pi_j = b_j - b_j p_{0i}(T)$. Note

that b_{js}^{p} can be expressed as $(b_{js}^{-}0)_{0i}$. Thus equation (4)

now reduces to

(5)
$$f = b - \sum_{n=0}^{b-1} (b-n) p_{ni}(T)$$

Equations (4) and (5) are valid for all $b_{js} \ge 1$. By definition $f_{ijs} = 0$ for $b_{js} = 0$, since no sticks of type j weapon can be dropped if none are carried.

As noted before, the random distribution of targets and consequent exponential inter-encounter times lead to a Poisson counting

process in determining the probabilities of encountering given numbers of targets; i.e., for type i targets appearing at a mean rate r,

$$p_{ni}(T) = e^{-r_i T} \frac{(r_i T)^n}{n!}$$

Table 11, Appendix III gives values of f_{ijs} for various combinations of b and r.T. This table greatly facilitates calculation of a "best load" from the feasible loads being considered.

To determine the "best load" for a given set of targets, find that individual weapon type j, for each target, which has the highest kill probability. The loads considered are then the different mixes of these weapons selected from a list of feasible loads for the aircraft.

For a given load, b_{js} will be known for each type weapon in the load and the encounter rate r_i will be estimated for each of the critical targets. The planned search time T will also be known for the mission. Thus using known values of k_{ijs} and estimated values of v_i , E can be calculated.

Consider first the case where weapons will be dropped individually (sticks of one only) and where each weapon type in the load will be used against only one type target. In other words, target types and weapon types are matched one to one. Since weapons are to be dropped singly, $b_{js} = b_{j1}$ for each weapon type j. To determine E for such a load, first determine b_{j1} and r_{i} T for each weapon-target type combination. Then obtain f_{ijs} from Table 11, and multiply this

value by k v. Let f k v = E . Add the resulting E 's to ijs i ijs ijs i ij ij ij set \mathbf{E} to get E for the load, i.e., $\mathbf{E} = \sum_{i=1}^{m} \mathbf{E}_{ij}$. (Note that the \mathbf{E}_{ij} 's are summed \mathbf{E}_{ij})

over i only, since the subscript j simply indicates that weapon which is most effective against target type i.) In the same manner, E may be determined for each load being considered; that load which yields the maximum E is then the "best load."

For this sub-optimal model, the extension of the problem to include sticks of two or more weapons is accomplished quite easily, since different size salvos or sticks of a given type weapon constitute different loads. For example, as noted previously, two 250 lb. bombs might be considered as a load consisting of two sticks of one bomb or as another load consisting of one stick of two bombs. With this in mind, E can be calculated for these loads in the same manner as previously outlined. In order to keep the problem hand workable, this approach assumes that each delivery of a given type weapon will consist of the same size stick. This assumption is made solely for determination of the aircraft loading prior to flying the mission. During the actual mission, however, it is quite likely that sticks of different sizes will be expended at different times. For example, a pilot might drop one type j bomb on the first truck he sees, a stick of three type j bombs on the second truck he sees, etc. Prior to the mission, however, this load of four type i weapons could have been evaluated as one stick of four, two sticks of two, or possibly as four sticks of one, since calculations for sticks of varying sizes quickly exceed the bounds of practical hand workability.

In addition to the case where targets and weapons are matched one to one, consideration must be given to the case where a given weapon type is optimal for more than one target type. If the assumption that a target sighted will be dropped on were adhered to, the expected number of type i targets killed would be proportional to the encounter rate r. However, in this situation, a concession is made to the fact that, early in the mission, a target of lesser value might be passed in hopes of encountering a more highly valued target later. Thus it is assumed that the expected number of type i targets killed is proportional to both the encounter rate, r_i , and the target value, v_i . The expected return from a given load is now calculated exactly as before with the following minor modification: For all targets type i against which weapon type j is most effective, calculate the E_{ij} 's exactly as in the simpler case (as if all type j weapons were to be expended on each type target). Then to approximate the particular expected value of type y targets killed by weapon type j, E is multiplied by the normalizing factor $N_y = \frac{r_y v_y}{\sum r_z v_z}$ where the summation

in the denominator is taken over those type i targets for which weapon type j is optimal. These expected values are then added to give E as before.

A logical extension to this hand approximation is the consideration of "secondary" weapons. A secondary weapon would be considered to be a weapon which has high kill probabilities against several targets, yet is optimal against none of them. There are certainly cases where such a weapon might give higher expected returns over the entire target spectrum than would a load consisting of only those weapons which are optimal for the individual targets. For example, consider a case in which there are two targets and three weapons to be used against these targets. The single stick kill probabilities for the individual weapons are as follows:

WEAPON	TARGET 1	TARGET 2
1	.09	.01
2	.01	.09
3	.08	.08

If the load were limited to those weapons which are optimal for individual targets, only weapons 1 and 2 would be considered. However, it is obvious that weapon 3 merits examination, and, in fact, it appears as if a pure load of weapon type 3 might possibly be the best load to use in this particular case. Calculation of E for such a secondary weapon is identical to that in the foregoing case in which one weapon is considered for use against more than one target.

This sub-optimal development may, at first glance, appear to be long and cumbersome. It must be realized though that in most actual situations all but a very few feasible loads will be eliminated from consideration prior to any calculation, and these will be examined only as far as they pertain to a small number of critical target types (normally not more than three or four). Thus it can be seen that relatively few calculations will actually be required and these will be of a trivial nature.

In order to clarify the use of the sub-optimal model, the following simple example is presented in detail.

Consider an armed reconnaissance mission with a planned search time of one hour. (Thus T=1 so that $r_iT=r_i$ for all targets).

Three target types are considered:

$$v_1 = 2$$
 $r_1 = 5$
 $v_2 = 1$ $r_2 = 3$
 $v_3 = 4$ $r_3 = 2$

The aircraft has two weapon stations and two types of weapons are considered, one more effective against target type 1, the other more effective against types 2 and 3 (see Table 2).

The possible loads are indicated in Table 1.

	WEAPON			
Load	Single	Stick of 2	Single	Stick of 2
A	1		1	
В	2			
C			2	
D	1	1		
E	1			1

TABLE 1
COMPOSITION OF POSSIBLE LOADS

j	s i	1	2	3
	1	.02	.03	.01
1	2	.06	.08	.03
2	1	.01	.05	.02
	2	.03	.12	.05

TABLE 2
KILL PROBABILITIES

Load A:
$$b = 1, b = 1$$

Target 1 - Weapon 1

$$f_{111} = .99326$$
 (from Table 11; $b_{js} = 1$, $r_{i} = 5$)

$$E = f k v = (.99326) (.02) (2) = .0397$$

Target 2 - Weapon 2

$$f_{221} = .95021 (b_{js} = 1, r_{T} = 3)$$

$$E_{22} = f_{221} k_{221} v_{2} = (.95021) (.05) (1) = .0475$$

Target 3 - Weapon 2

$$f_{321} = .86466$$
 (b = 1, r T = 2

$$E_{32} = f_{321}^{k}_{321}^{v}_{3} = (.86466) (.02) (4) = .0692$$

Since weapon type 2 is optimal for two target types, E $_{22}$ and E $_{32}$ must be normalized.

$$N_{222} = (r_2 v_2 / r_3 v_2 + r_3 v_3) E_{22} = (3/11) (.0475) = .0129$$

$$N_3 E_{32} = (8/11) (.0692) = .0608$$

Thus for load A, E = .0397 + .0129 + .0608 = .1134

Load B:
$$b_{11} = 2$$

$$f_{111} = 1.95283$$
 (b = 2, r T = 5)

$$E_{11} = E \text{ (Load B)} = (1.95283) \text{ (.02) (2)} = \underline{.0724}$$

Load C:
$$b = 2$$

$$f_{221} = 1.75106$$
 ($b_{js} = 2$, $r_{i}T = 3$)

$$N_2E_{21} = (3/11) (1.75106) (.05) (1) = .0240$$
 $f_{321} = 1.45866 (b_{js} = 2, r_{i}T = 2)$
 $N_3E_{332} = (8/11) (1.45866) (.02) (4) = .0846$
 $E(Load C) = .0240 + .0846 = .1086$

Load D: b = 1 (Note that this is almost the same as Load B, but
12

now the two weapons are considered as one stick of two vice two sticks

of one).

f = .99326

$$112$$

E = E(Load D) = (.99326) (.06) (2) = .1192
Load E: b = 1
22

$$f_{222} = .95021$$

$$N_{2}E_{22} = (3/11) (.95021) (.12) (1) = .0312$$

$$f = .86466$$

$$N_3 E_{22} = (8/11) (.86466) (.05) (4) = .1252$$

$$E \text{ (Load E)} = .0312 + .1252 = \underline{.1564}$$

For the loads considered, the maximum E is .1564. Therefore the "best load" is load E which consists of one stick of two type 2 weapons. Note that if sticks of one only (case 1 in the model development) had been considered, the "best load" would have been load A with one of each type weapon yielding E = .1134.

The foregoing sub-optimal method is limited in that it considers search time as a whole rather than a time-stepped firing doctrine as

in the dynamic programming method. Thus those added values which might be generated by strategies which vary with mission time are lost in this formulation. Consequently the expected kill values will generally be much lower than those of the dynamic program formulation, and the "best load" generated will not always be the optimum load. However, the load obtained by this method should, if expended in accordance with the optimum firing doctrine of the dynamic program, be one of the more highly valued loads.

3. SAMPLE RESULTS

OPTIMAL LOAD AND FIRING STRATEGY DETERMINATION

For a mission time of 15 minutes, seven weapons from which to choose, and values of the other input parameters as listed in Table

3, an optimal loading has been determined using the "BESTLD" program of Appendix II, with the following inputs:

Weapon Type: 1 2 3 4 5 6 7

Weapon Weight: 265 300 510 600 675 950 1975

Number of racks = 5

Max payload = 5,000 lbs.

Number of target types = 4

Mission time = 0.25 hours

Delta T = .010 hours (This is the Δt of equation (3)).

TARGET TYPE	ENCOUNTER RATE 30.00000 1.00000	VALUE 1.00 25.00
3	.10000 3.0000	60.00 5.00

TABLE 3
KILL PROBABILITIES

These weapons' weights and aircraft payload and rack capacities result in approximately 400 possible loadings which must be considered. From amongst these 400, Table 4 lists the five loads with the greatest expected returns and for comparison the loading yielding the least. A loading designated 1200002 denotes one weapon of type one, two weapons of type two, zero weapons of types three through six, and two weapons of type seven.

LOAD NO.	LOADING	EXPECTED RETURN
1	0300002	1.29514
2	1200002	1.25048
3	0100102	1.22353
4	1000102	1.20670
5	3000002	1.20409
381	2011010	.38069

TABLE 4
LOADINGS AND EXPECTED RETURNS,
DYNAMIC PROGRAMMING METHOD

The expected returns from the optimal load are over three times higher than those of the worst. In fact the twentieth best load, 0020300, had expected returns of only .77700. Thus the optimal loading yields returns more than 65% better than a loading which is better than 95% of the possible loads.

The above inputs and constraints have also been used in the suboptimal model to generate a "best load." Weapon 7 is the best single
weapon against targets 1 and 2, and weapon 5 is best against targets
3 and 4. Thus the only loads considered in this formulation were
0000500, 0000401, and 0000102. Table 5 lists six possible load considerations (including stick options) and the expected returns from each.

Loading							E	
0000102	(Weapon	7	used	as	a	stick of 2)	.5766	
0000401	(Weapon	5	used	as	а	stick of 4)	.4458	
0000102	(Weapon	7	used	as	2	sticks of 1)	.3824	
0000500	(Weapon	5	used	as	а	stick of 5)	.3740	
0000401	(Weapon	5	used	as	4	sticks of 1)	.3320	
0000500	(Weapon	5	used	as	5	sticks of 1)	.1432	

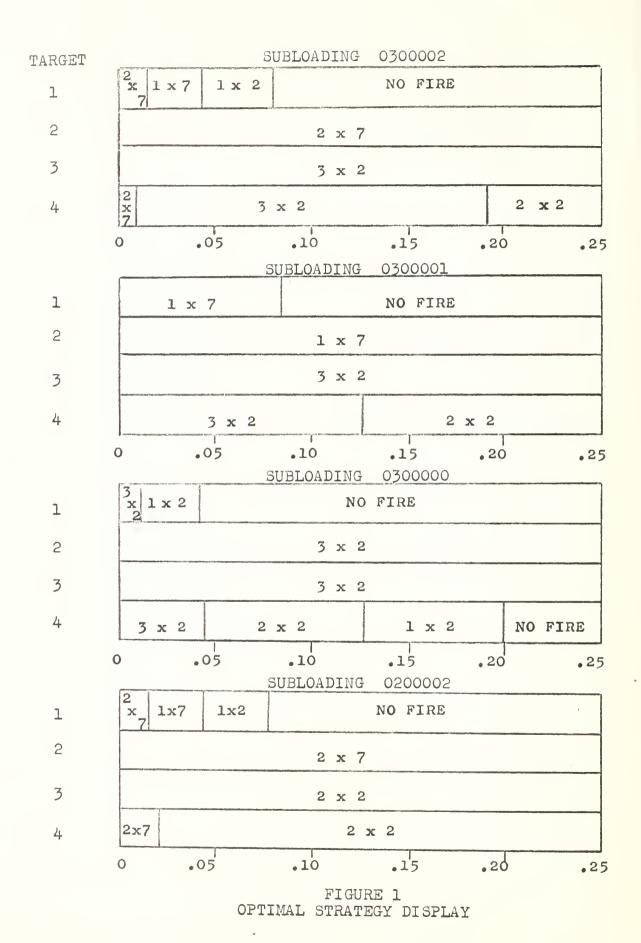
TABLE 5
POSSIBLE LOADS AND EXPECTED
RETURNS, SUB-OPTIMAL METHOD

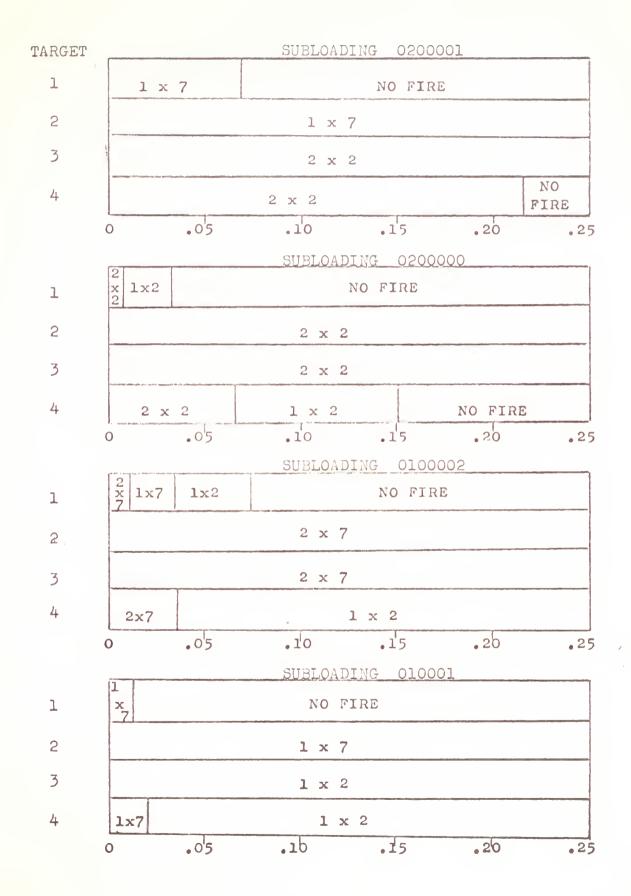
The "best load" using the sub-optimal hand method is 0000102 considering weapon 7 as one stick of two. Although only the principal weapon for each target has been considered, note that weight constraint still allows the loading of one more of the lighter, less effective weapons (weapon 2 rather than weapon 1 since weapon 2 is dominant over the entire target spectrum). The load thus generated corresponds to the third most valuable dynamic programming load, and, if used in accordance with the dynamic programming firing policy, it would be expected to yield a return of greater than 90% of that given by the actual optimum load.

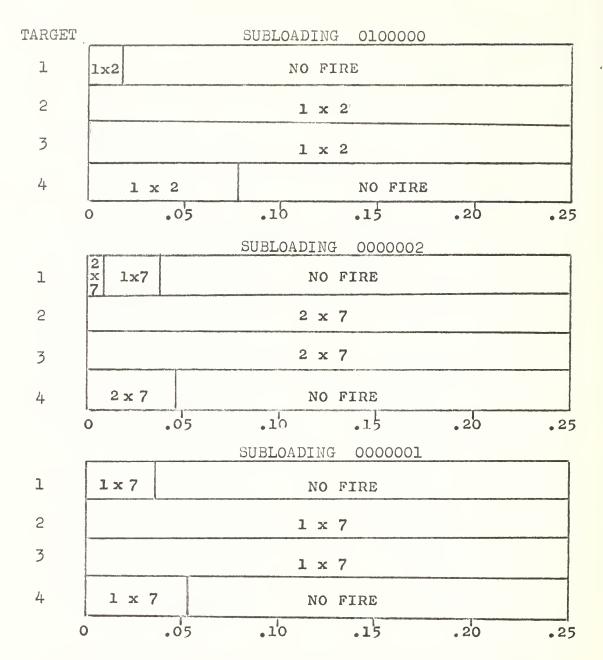
With the best loading as generated by either method, the optimal strategy can be listed using the "POLICY" program of Appendix II. The results of this determination for loading 0300002 are shown in Figure 1 along with a format for displaying the strategy compactly. The time axis is mission time remaining and entries such 2x7 denote that two weapons of type seven should be expended.

As an example, as the mission begins, the subloading is 0300002.

If the first target which is met is a type one, and is encountered with







(The abscissa in these figures is the time remaining in which to search for and attack targets.)

.22 hours remaining, the optimal strategy indicates that no weapons should be fired. If the next target which is encountered is a type four at time .17, the subloading still being 0300002, a drop of three weapons of type two is called for. The sub-loading now is 0000002. If a type three target is now met, both of the remaining type seven weapons should be dropped.

It is of interest to note the changes which occur over time in the optimal weapon to drop when a target is met. With the loading 0300002 and target type one, with more than .08 hours remaining, no weapons should be expended. With .045 to .08 hours remaining, the optimal strategy calls for the expenditure of one weapon of type two. Weapon type seven however has a higher kill probability against type one targets! The type seven weapons at this point are being saved for possible future encounters with targets of type two and four, both of which are more valuable and occur frequently enough so that there is a significant probability of an encounter in the time remaining. Although weapon type two is the most potent against type three targets -the most valuable target type- the probability of meeting a type three target is rather small because of the low encounter rate of 0.10. It is for this reason that type two weapons are not being conserved. Later in the mission however, the optimal weapons to drop on a type one target are type sevens. At this point in time the probability of future encounters with more valuable targets is remote.

Since the ultimate objective is the maximization of the expected returns for the mission, a consideration might be made for lengthening the search time at the expense of lower payload and weapon station availability. That is, perhaps an additional tank of fuel can be

carried on one of the aircraft's weapon racks, thus reducing the number available on which ordnance can be carried and also reducing the weapons payload, but increasing the search time.

Table 6 lists the expected returns for the five best loads generated when the search time has been increased to 0.50 hours, the payload decreased to 4000 lbs., the number of weapon racks decreased to four, and the other input parameters unchanged from those of Table 3.

Ī	OADING	3	EXPECTED	RETURNS
(00000	2	1.788	03
(000030	1	1.421	84
(10020	1	1.300	25
(01020	1	1.297	48
(000400)	1.2880	38
		TA	ABLE 6	
BEST	LOADS	AND	EXPECTED I	RETURNS

Since the expected value of targets killed for the longer mission is significantly better than that for the shorter, 1.78803 compared to 1.29514, it is clearly preferrable to make the trade-off of gas for weapons.

An interesting feature of the kill probability matrix of Table 3 is the preferred weapons with which to attack the most valuable targets. Weapon type seven is the most potent against type two targets while weapon type five is preferred against type three targets. Target type three, although the most valuable, has a very low encounter rate. This results in the fact that weapon type five does not appear frequently in the loadings which yield high expected returns for short missions. Letting the maximum payload equal 4000 lbs. and the number of weapons stations equal four, the preferred loadings of these two

weapons alone can be examined as a function of search time. The three loadings which must be considered are 02, 31, and 40, where now, 02 denotes a load consisting of zero weapons of type five and two weapons of type seven. Calculations show that an initial loading of 02 is optimal for mission times of less than 1.6 hours. For mission times of 1.6 to 8.9 hours, loading 31 is optimal, and finally for T > 8.9 hours, loading 40 is preferred.

COMPARISON OF EXPECTED RETURNS FROM NON-OPTIMAL FIRING STRATEGIES

The gains in kill potential realizable through the utilization of the optimal firing doctrine can be demonstrated by comparisons with the expected returns from other strategies. As an example, consider the following case with non-optimal strategies:

Let:

Number of weapons = 1

Operating time = 1 hour

No. of target types = 2

 $v_1 = 3$

 $v_2 = 4$

Strategy A: Expend the weapon on the first target which is encountered.

 E_A = the expected returns from strategy A.

Strategy B: Expend weapon only if a target of type 2 is encountered. (Note that target type 2 is the more valuable).

 $\mathbf{E}_{\mathbf{p}}$ = the expected returns from strategy B.

T: The random variable describing the time to first encounter with target type i. (T is exponentially distributed with mean 1/r;.)

$$E_{A} = P_{1}v_{1}Prob. (T_{1} < min (t_{2}, 1 hour)) + P_{2}v_{2}Prob. (T_{2} < min (t_{1}, 1 hour))$$

$$E_B = P_2 v_2 \text{ Prob. } (T_2 < 1 \text{ hour})$$

$$E_B = P_2 v_2 \int_0^1 r_2 e^{-r_2 t_2} dt_2$$

For several values of the parameters p_i and r_i , Table 7 shows the results of calculations of the expected returns using either of these strategies or the optimal.

		cted Val			
		rgets Ki	% Gain (
in the second		Strat.	Optima1	Ove	i i
	A	В	Strat.	A	В
p = .3				To the common of	
p ₂ = .4	.963	1.011	1.270	32%	26%
r = 10				STORY OF THE STORY	• •
r = 1 2		STEE Promitting of the Control of th		and Chiming Work Agents	
p ₁ = .3				Control of the Contro	
p ₂ = .4	.916	.354	1.018	11%	185%
r = 10		• 337		11/0	100/6
r = .25	To Wind the man of the Control of th				
p = .1					
p ₂ = .4	.414	1.011	1.064	150%	5%
$r_1 = 10$					
r ₂ = 1	To the second se				

TABLE 7

GAINS IN EXPECTED VALUE OF TARGETS KILLED ACHIEVED BY UTILIZING THE OPTIMAL FIRING STRATEGY

These results are representative of those obtainable using other values for the input parameters. Generally, when one of these non-optimal strategies approaches the optimal in expected returns, the other yields very poor results. When the non-optimal strategies yield roughly equivalent returns, the optimal dominates both by a significant amount; this is demonstrated in the first example of the table by gains of 26% and 32% through use of the optimal strategy.

Comparisons can be made in this way with any conceivable strategy. It seems reasonable too that techniques which consider the target values and encounter rates could yield returns comparable to those achievable through use of the optimal strategy. The determination of these strategies however would probably involve calculations no less complex than those of the dynamic program.

SENSITIVITY OF EXPECTED RETURNS TO TARGET VALUES

A complete parametric analysis is beyond the scope of this paper; however, some further comment must be made about target values. The developments of both the dynamic programming model and the hand approximation are based upon the assumption that a ratio scale of values can be assigned to all possible targets. Such a determination is completely subjective and must rely solely on the judgement of experienced planners. Quantitative target values might be determined by presenting a list of targets to several military commanders and asking them to rate these targets according to military value. Targets could be assigned numbers consistent with their importance relative to a target of least value. The results of such a large number of opinions could then be analyzed by methods of sensory psychophysics in order to assign representative

values to all targets. Methods for accomplishing such an analysis have been reviewed and consolidated by S. S. Stevens. 1

The above method will obviously generate strategic values. These will certainly not be valid for every tactical situation (e.g., a tank about to overrun friendly troops is more important than a railroad bridge at that moment even though the bridge may have a higher strategic value). A strategic scale will, however, provide a starting point -- a base which can be adjusted to meet specific tactical situations.

An analysis of the effects on the expected returns of incorrect selections of target values can be made rather easily for the simple case of a single weapon and two target types. Assume the "true" ratio of the target values is V_r^* . Now suppose that an erroneous ratio, V_r^* , has been used to calculate the strategy. The expected returns which result from the utilization of this strategy - these returns being computed using the true value ratio, V_r^* - can be compared with the maximum of the expected returns, this maximum being obtained when the strategy derived using V_r^* is utilized. Define:

$$v_r = v_2/v_1$$

and let the mission time = 1 hour. With a single weapon, a strategy consists merely of a cross-over time, the time after which the least valuable target should be attacked if met. These times can be calculated from:

$$t = \frac{1}{r_1 + r_2} \ln \frac{r_1 p_1 + r_2 p_2 v_r}{r_2 (p_2 v_r - p_1)} \text{ if } v_r > 1$$

$$t = \frac{1}{r_1^+ r_2} \ln \frac{r_1 p_1 + r_2 p_2 V_r}{r_1 (p_1 - p_2 V_r)} \quad \text{if } V_r < 1$$

$$t = \infty$$
 if $v_r = 1$

¹S.S. Stevens, "A Metric for Social Concensus," Science, vol. 151, 4 February 1966, pp. 530-541.

Using these cross-over times then, the expected returns, E, are calculated from:

$$\begin{split} \mathbf{E} &= \mathbf{p}_{2} \mathbf{V}_{r} \star \frac{\mathbf{r}_{2}}{\mathbf{r}_{1}^{+} \mathbf{r}_{2}} (1 - \mathbf{e}^{-(\mathbf{r}_{1}^{+} \mathbf{r}_{2}^{2})}) + \frac{\mathbf{p}_{1}^{r}_{1}}{\mathbf{r}_{1}^{+} \mathbf{r}_{2}} (1 - \mathbf{e}^{-(\mathbf{r}_{1}^{+} \mathbf{r}_{2}^{2})}) & \text{if } \mathbf{t} \geq 1 \text{ hour} \\ \mathbf{E} &= \mathbf{p}_{2} \mathbf{V}_{r} \star (1 - \mathbf{e}^{-\mathbf{r}_{2}^{(1-t)}}) + \mathbf{e}^{-\mathbf{r}_{2}^{(1-t)}} (1 - \mathbf{e}^{-(\mathbf{r}_{1}^{+} \mathbf{r}_{2}^{2})}) \frac{\mathbf{r}_{1}}{\mathbf{r}_{1}^{+} \mathbf{r}_{2}}) \\ &+ \mathbf{p}_{1} \mathbf{e}^{-\mathbf{r}_{2}^{(1-t)}} (1 - \mathbf{e}^{-(\mathbf{r}_{1}^{+} \mathbf{r}_{2}^{2})}) \frac{\mathbf{r}_{1}}{\mathbf{r}_{1}^{+} \mathbf{r}_{2}} & \text{if } \mathbf{t} \leq 1 \text{ hour, and } \mathbf{V}_{r} > 1 \\ \mathbf{E} &= \mathbf{p}_{1} (1 - \mathbf{e}^{-\mathbf{r}_{1}^{(1-t)}}) + \mathbf{e}^{-\mathbf{r}_{1}^{(1-t)}} (1 - \mathbf{e}^{-(\mathbf{r}_{1}^{+} \mathbf{r}_{2}^{2})}) \frac{\mathbf{r}_{1}}{\mathbf{r}_{1}^{+} \mathbf{r}_{2}}) \\ &+ \mathbf{p}_{2} \mathbf{V}_{r} \star \mathbf{e}^{-\mathbf{r}_{1}^{(1-t)}} (1 - \mathbf{e}^{-(\mathbf{r}_{1}^{+} \mathbf{r}_{2}^{2})}) \frac{\mathbf{r}_{2}}{\mathbf{r}_{1}^{+} \mathbf{r}_{2}} & \text{if } \mathbf{t} \leq 1 \text{ hour, and } \mathbf{V}_{r} \leq 1 \end{split}$$

Letting:

Table 8 lists the results for several possible values of V *.

While the scope of these investigations of value sensitivity is obviously limited, it appears that over-estimation of the value of targets of higher worth might impose less risk than under-estimation. Further examination in the area of sensitivity where several weapon types are employed simultaneously would be necessary before conclusive results could be stated.

	If V * is misestimated by a factor of z, the expected returns decrease by y.						
V *	z=10 y	z=5 y	z=1/2 y	z=1/10 y			
1	28%	22%	5%	8%			
2	8%	6%	10%	37%			
3	4%	3%	5%	53%			
5	2%	1%	3%	67%			
7	1%	1%	1%	66%			
10	1%	<.5%	1%	40%			

TABLE 8 PERCENTAGE DECREASE IN EXPECTED RETURNS DUE TO MISESTIMATION OF $\mathbf{V_r}^{\star}$

4. CONCLUSIONS AND RECOMMENDATIONS

A method for determining an optimal initial loading for missions against targets of opportunity has been developed. As a by-product of this load determination, an optimal strategy for the sequential expenditure of weapons has been found. The gains in the expected value of targets killed, when the initial loading is optimal and when an optimal strategy is employed, have been shown to be significant when compared with initial loadings and firing doctrines derived by the application of simple intuition and judgement to kill probabilities, target values, and encounter rates.

The determination of an optimal loading and expenditure policy, however, are at present of such complexity as to require the use of a high speed computer in all but the most trivial cases. For those operational situations in which a computer will not be available, a hand-calculable approximation method for the determination of a best initial loading has also been developed, which, though sub-optimal, has been found to generate loads which yield high expected returns.

Either of the above methods should yield significantly better results than present policies which prescribe that aircraft be loaded for targets which are assumed known or that several aircraft each be given different loads in order to cover an unknown spectrum of targets.

The models presented in this paper assume that weapons can be loaded in any combination which satisfies rack and weight constraints, and that the pilot has complete freedom of choice as to the sequence in which weapons can be expended. For most attack aircraft these assumptions are not always true. Restrictions on unsymmetrical loadings

are one limiting factor. Others relate to the mechanics of multiple bomb racks. The models should be expanded to account for these aspects of the problem.

A vital assumption in the developments of both models was that inter-encounter times would be distributed exponentially. It is recommended that an analysis of operational data be made to ascertain the validity of this supposition.

It seems reasonable to assume that a list of all possible types of targets of opportunity, for a spectrum of scenarios, would be small enough - certainly not larger than 50 - that a thorough analysis, as suggested in Section 3, could be made of their relative values. Such an analysis would indicate general strategic values which could then be adjusted for specific tactical scenarios. Furthermore, additional investigations similar to that outlined in Section 3 should be undertaken to study the sensitivity of the optimal load and strategy calculations to these values.

The hand approximation method is rather crude in form. It is recommended that this model be examined further in an attempt to refine the calculations and to extend the model to cover a wider variety of cases in a more complete fashion.

The computer programs of Appendix II were written solely for the computations used in this thesis and should be improved upon before extensive investigations are made of the dynamic programming treatment of the problem. The four target-seven weapon example required two hours and fifteen minutes on a CDC-1604 computer for the determination of an optimal load. This is testimony to the inefficiency of the "BESTLD" FORTRAN program.

The problem of determining an optimal initial loading is similar to the classic "flyaway-kit" problem of dynamic programming. In the strike aircraft loading problem the additional complication is present that the costs of shortages are not constant but depend on the initial loading. It would seem though that research into solution of the strike aircraft problem by this method should be fruitful. Very good approximations to an optimal initial loading should be obtainable very rapidly in this way.

Although the "POLICY" FORTRAN program computes the optimal strategy with relative ease, usually less than four minutes is required on a CDC-1604 computer, it is recommended that methods of approximation be developed for which a strategy can be calculated without the aid of a high speed computer.

Bellman, R. E. and Dreyfus, R. E., Applied Dynamic Programming, (Princeton, New Jersey: Princeton University Press, 1962), pp. 42-43.

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APPENDIX I

ENCOUNTER RATES

In a combat situation, many targets (especially "targets of opportunity") may be thought of as being distributed randomly throughout a given area, in that the pilot flying the mission does not know, beforehand, where or when a target will be encountered. This may be thought of as the classical situation in which it may be assumed that, in any small time interval dt, the probability that a target is encountered is proportional to dt and is independent of any previous encounters.

Over a period of time, photo or visual reconnaissance can give an average density of targets in a given area. From the attack aircraft speed and the average search sweep width (width of a path in which the pilot could be expected to see a target) the area swept per unit time can be obtained. By multiplying the average type i target density by the average sweep area per unit time, an estimate of the average encounter rate r, is obtained for target type i.

The average encounter rate developed above can be used to generate the distribution of inter-encounter times.

Assumptions:

Probability of one encounter in $dt = r_i dt + h(dt)$ where h(dt) approaches zero as dt approaches zero.

Probability of two or more encounters in dt = h(dt)

Probability of no encounters in $dt = 1-r_i dt + h(dt) p(t) dt = prob.$ (t \le first encounter time \le t + dt) = dP(t)

$$P(t) = \int_{0}^{t} p(T) dt = prob.(encounter time \le t)$$

$$Q(t) = 1 - P(t)$$

Thus if t is the encounter time, and t is any arbitrary time in the mission,

$$Q(t+dt) = \text{prob.}((t > t) (\text{no encounter in dt}))$$
$$= Q(t) (1-r, dt + h(dt))$$

Subtracting Q(t) from both sides, dividing through by dt and taking the limit as dt approaches zero gives dQ(t)/dt = -Q(T)rRearranging gives dQ(t)/Q(t) = -r dt

Integrate both sides to get $ln Q(t) = C-r_{i}t$

Thus $Q(t) = Ae^{-r}i^t$

But Q(0) = 1 since there can certainly be no encounters in zero search time. Therefore A = 1 and $Q(t) = e^{-r}i^t$ and consequently $P(t) = 1 - e^{-r}i^t$ and $p(t) = r_i e^{-r}i^t$

Thus it is seen that the inter-encounter times are exponentially distributed. It also follows naturally that the probability distribution on the number of targets type i encountered during a mission of length T is Poisson, 1 i.e.,

prob. (N(T) = n) =
$$(r_i T)^n e^{-r_i T}/n!$$

and the expected no. $\overline{N}(T) = r T$ (This is obtained from the observations).

Parzen, E., Modern Probability Theory and Its Applications, (New York: John Wiley and Sons, 1960) p. 253.

APPENDIX II

BESTLD AND POLICY FORTRAN PROGRAMS

()	ROGRAM BESTLD IMENSION XLAM(10),P(8,8,10),V(10),X(512),F(512),N(8),W(8) ISUBS(512) XIMUM OF 8 TARGET TYPES, 8 WEAPON TYPES, AND 10 WEAPON RA EAD 551,L,LL,WM ORMAT (212,F10,0)	000000 000010 000020 000030 000040
υU	L IS THE NUMBER OF WEAPON RACKS, LL IS THE NUMBER OF WEAPON TYPES, WM IS THE MAXIMUM PAYLOAD OF THE STRIKE AIRCRAFT READ 552,(W(I),I=1,8)	000000000000000000000000000000000000000
	MAT (8F6.0)	060000
, ()	W(I) IS THE WEIGHT OF WEAPON TYPE I, IF THE NUMBER OF WEAPON TYPES IS LESS THAN 8, SET W(LL+1), W(LL+2),,W(8) EQUAL TO WM+1.	000100
	D 200, M,	000120
	FORMAT (15,2F5.0)	000130
	S THE NUMB	000140
	T IS THE MISSION TIME RFAD 202 (XLAM(I) V(I) 1=1 W)	000150
. 4	ORMAT (2F5.	000170
	XLAM(I) IS THE ENCOUNTER RATE WITH TARGET TYPE I,	000180
()	I) IS TH	000190
	0 825 K=	000200
	0 825 JN=1,	000210
	READ 826, (P(I,JN,K),I=1,L)	000220

 \cup \cup

000720 000560 000570 000610 000630 000640 000650 099000 000670 089000 069000 007000 000710 000730 000740 000750 000770 000780 0007000 008000 000810 000820 000830 000840 0088000 000580 000590 009000 000620 090000 000860 .GE. AMIN) 503,506 IF (NLOAD .LE. L) 823,501 IF (NLOAD .EQ. L) 504,501 IF(WLOAD .LE. WM) 504,501 WLOAD=WLOAD+J(I)*W(I) NLOAD = NLOAD + J(I) DO 501 J2P=1,J2T DO 501 J3P=1,J3T DO 501 J4P=1,J4T DO 501 J5P=1,J5T DO 501 J6P=1,J6T DO 501 J7P=1,J7T DO 501 J8P=1,J8T J3T = J2T - J(2)DO 502 I=1,8 J4T=J3T-J(3) J6T=J5T-J(5) J7T=J6T-J(6) J5T = J4T - J(4)J8T = J7T - J(7)J(5) = J5P - 1J(6) = J6P - 1J(2) = J2P - 1J(3) = J3P - 1J(4) = J4P-1J2T=9-J(1) J(7) = J7P - 1J(1) = J1P - 1J(8) = J8P - 1(WLOAD WLOAD=0.0 NLOAD = 0NIP 823 502 506 503 504

```
001070
000870
       000880
              000890
                     006000
                            000910
                                   000020
                                          000030
                                                 076000
                                                        0000950
                                                               096000
                                                                     000070
                                                                             086000
                                                                                          001000
                                                                                   066000
                                                                                                 001010
                                                                                                         001020
                                                                                                                001030
                                                                                                                      001040
                                                                                                                              001050
                                                                                                                                     001060
                                                                                                                                                         001000
                                                                                                                                                                001100
                                                                                                                                                                               001120
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                                                                                                                                                                        001110
                                                                                                                                                                                                    10**4
                                                        <u>م</u>
                                                        Z
                                                        yk.
                                                                                                                                                                                                    3%
                                                       N7P
                                                        깛
                                                                                                                                                                                                     +
                                                       M6P
                                                                                                                                                                                                   10**5
                                                                                                                                                                                                           +
                                                        ×
                                                                                                                                                                                                          10
                                                       NSP
                                                                                                                                                                                                    эķс
                                                                                                                                                                                                          υķε
                                                        3/4
                                                                                                                                                                                                   * 10**6+
                                                       NAP
                                                        3/0
                                                                                                                                                                                                          10 *
                                                       0.EN
                                                                                                                                                                                                   +12
                                                                                                                                                                                                         *
                                                                                                 1,N2P
                                                                                                                1,N3P
                                                                                                                             1,N4P
                                                                                                                                           1,N5P
                                                                                                                                                                                     =1,N8P
                                                                                                                                                                       =1,N7P
                                                       = NIP * NZP
                                                                                                                                                         1000 NN6=1,N6P
                                                                                                                                                                                                   0 × × 7
                                                                                                                                                                                                                        JPROD
                                                              I=1,IPROD
                                                                                                                                                                                                          +
                                                                                                                                            11
                                                                                    li
                                                                                                                П
                                                                                                                              11
                                                                                                  11
                                                                                                                                                                                                          + 15 * 10**3
                                                                                                                                                                      1000 NN7
                                                                                                                                                                                     1000 NN8
                                                                                                                                                                                                   JPROD = 11 *
                                                                                                               1000 NN3
                                                                                                                                           1000 NN5
                                                                                                                                                                - 9NN =
                                                                                  DO 1000 NN1
                                                                                                 1000 NN2
                                                                                                                             1000 NN4
                                                                                                                                                 - SNN =
                                                                                                        - NN =
                                                                     0.0
                                                                                          - INN =
                                                                                                                     = NN3 -
                                                                                                                                    = NN4-1
                                                                                                                                                                              = NN7-1
                                                                                                                                                                                            = NN8-1
                                  (7)
                                          (8)
      )(3)
             )(4)
                            1(6)
)(2
                    315
                                                                                                                                                                                                                        ISUBS(JJ)
                                                 9
                                                              DO 101
                                                                       ŧI
                                                      IPROD
                            11
                                   П
                                          11
 П
                                                                                                                                                                                                                 = ff
                                                                     ( I ) X
                                                                           17=0
                                                66 I
      N3P
                    NSP
                           N6P
                                  N7P
                                         N8P
             N4P
                                                                                                                             00
                                                                                                                                                         00
                                                                                                                                                                                     00
                                                                                                 00
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                                                                                                                      23
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                                                                                                                                    7 I
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001240
                                                       0.01230
                                                                                                       001270
                                                                                                                                                                                                                           001360
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                                                                                                                                                                                                                                                                                                                                                             001460
                                                                                                                                                                                                                                                                                                                                                                                                  001490
                                        001220
                                                                               001250
                                                                                            001260
                                                                                                                     001280
                                                                                                                                  001290
                                                                                                                                               001300
                                                                                                                                                           001310
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                                                                                                                                                                                                                                                                                                                                  001440
                                                                                                                                                                                                                                                                                                                                                                         001470
                                                                                                                                                                                                                                                                                                                                                                                     001480
                                                                                                                                                                                                                                                                                                                                                                                                               001500
                                                                                                                                                                                                                                                                                                                                               EXPENDED
                                                                                                                  THIS IS THE TIME REMAINING IN WHICH TO OPERATE
                                                                                                                                                                                                                                                                                                                                              TYPE 14
                                                                                                                                                                                                 * 10**(8-IY)
                                                                                                                                                                                                                                                                                                                                             JI4 IS THE NUMBER OF WEAPONS OF
                                                                                                                                                                                                                                                                                                                                                                                                             = ILOAD - JI4*10 **(8-14)
                                                   TEMP = 1.0 - DELTA* SUMLAM
                                                                                                                                                                                                                                                                                                                                                                                               = P(JI4,I4,I3) *
                                                                                                                                                                                  N(IY) = ITEM/10**(8-IY)
                                       SUMLAM=SUMLAM+ XLAM(I)
                                                                                                                                                                                                                                                                                                      IF (NL-1) 105,105,107
                                                                                                                                                                                               ITEM = ITEM - N(IY)
                                                                                                                                DO 103 I2 = 2, IPROD
ISUBS (IPROD)
                                                                                                                                                                                                                                                                                                                                                          IF(JI4) 640,640,641
                                                                                                     TIME = (II-1) * DELTA
                                                                                                                                                                                                             TERM1= TEMP *X(12)
                                                                                         DO 102 Il=2,NDIV
                                                                             NDIV = XNDIV + 1
                                                                                                                                            ILOAD = ISUBS(I2)
                                                                XNDIV = T/DELTA
                                                                                                                                                                     DO 625 IY =1,8
                                                                                                                                                                                                                                                                                         NL = N(14) + 1
                                                                                                                                                                                                                                                                                                                  DO 106 I5=1,NL
                                                                                                                                                                                                                                      DO 104 I3=1,M
                                                                                                                                                                                                                                                                14 = 198
                         DO 100 I=1,M
                                                                                                                                                         ITEM = ILOAD
              0.0 =
                                                                                                                                                                                                                                                                                                                                                                     FIRST = 0.0
                                                                                                                                                                                                                        TERM2=0.0
                                                                                                                                                                                                                                                                                                                               JI4 = I5-1
                                                                                                                                                                                                                                                                                                                                                                                   GO TO 642
              SUMLAM
                                                                                                                                                                                                                                                               DO 105
 LOAD =
                                                                                                                                                                                                                                                   V=0.0
                                                                                                                                                                                                                                                                             A=0.0
                                      100
                                                                                                                                                                                               625
                                                                                                                                                                                                                                                                                                                  107
             58
                                                                                                                               721
                                                                                                                                                                                                                                                                                                                                                                      049
                                                                                                                                                                                                                                                                                                                                                                                                641
                                                                                                                                                                                                                                                                                                                                                                                                            642
                                                                                                                                                                                                                                                                                                                                             \cup
```

	0 108 16 = 1.IP	7 2
	F (JSUBS - ISUBS	0152
109	7 = 16	0153
	60 TO 110	0154
	ONTINUE	0155
110	ECOND=X(I7)	0156
	=FIRS	0157
	F (Z-A)106,	0158
$\overline{}$	2 =	0159
106	ONTINUE	0160
	<u> </u>	0161
	✓	0162
	ONTINUE	0163
0	ERM2 = TERM2 + Y	0164
	(I2)=TERM1 + TE	0165
103	ONTINUE	0166
	F (199 - 11	0167
129	6I = 66	0168
	RINT 320, (N(IY), IY=1,8),T	0169
\sim	ORMAT (1X,1	0170
2	O 113 IT=2, IPRO	0171
113	(II) = F(II)	0172
0	ONTINCE	0173
	RINT 88	0174
888	ORMAT (///	0175
	F (F(IPRO	0176
141	Z = F(IPRO)	0177
	5 = 10*LOA	0178
501	ONTINUE	0179
	RINT 666,K	0180
999	ORM	0181
	F9.51	001820
	FND	0183

000170(•100) = •5633		= .56935 = .76357 = .94750 = 1.12276	.150) .200) .250)	00000120(00000120(00000120(
-----------------------	--	---	-------------------------	-------------------------------------

42	3714	•47282	5709	9999
п	н	н	11	н
.050)	00	.150)	00	250
00000310	000310	00000310	000031	000031

BESTLD SAMPLE OUTPUT

TABLE 9

	0029 0030 0031
PROGRAM POLICY DIMENSION XLAM(10),P(8,8,10),V(10),X(512),F(512),N(8), I ISUBS(512),KFIRE(9,10),JFIRE(9),LFIRE(9) READ 200,M*N1,N2,N3,N4,N5,N6,N7,N8,DELTA,T 200 FORMAT(913,2F5.0) M IS THE NUMBER OF WEAPONS OF TYPE 1, ETC. IF THERE ARE LESS THAN 8 WEAPON TYPES,SET N8 = 0.ETC. DELTA IS DELTA T, IS THE MISSION LENGTH. READ 202,(XLAM(1),V(1),1=1,M) 202 FORMAT (2F5.0) XLAM(1) IS THE VALUE OF TARGET TYPE 1. READ 202,(XLAM(1),V(1),1=1,M) 202 FORMAT (2F5.0) XLAM(1) IS THE VALUE OF TARGET TYPE 1. READ 777-L.LL READ 777-L.LL READ 777-L.LL READ 777-L.LL READ 777-L.LL READ 777-L.LL READ 826 (P(1,JN,K),1=1,L) DO 825 M=1,M DO 825 M=1,M DO 825 W=1,M DO 825 W=1,M DO 825 W=1,M NO 825 W=1,L READ 826 (P(1,JN,K),1=1,L) RAGET OF TYPE K. RAGET OF TYPE K. RAGET OF TYPE K. RAPONS OF TYPE JN WILL NAP=N4+1 N4P=N4+1 N5P=N4+1 N6P=N4+1 N6P=N6+1	+ 2 N = + OC = OC

```
0000370
                                                                      000380
                                                                                                                                                                    000460
                                                                                                                                                                                                                               000510
                                                                                                                                                                                                                                          000520
                                                                                                                                                                                                                                                                             000550
                                                                                                                                                                                                                                                                                          000560
                                                                                                                                                                                                                                                                                                     000570
                                                                                                                                                                                                                                                                                                                000580
                                                                                                                                                                                                                                                                                                                             000590
                                                                                                                                                                                                                                                                                                                                        009000
                                                                                                                                                                                                                                                                                                                                                    000610
                                                                                                                                                                                                                                                                                                                                                               000620
             000330
                        000340
                                               000360
                                                                                  000390
                                                                                              000400
                                                                                                         000410
                                                                                                                      000420
                                                                                                                                 000430
                                                                                                                                             000440
                                                                                                                                                         000450
                                                                                                                                                                                000470
                                                                                                                                                                                           000480
                                                                                                                                                                                                       064000
                                                                                                                                                                                                                  000500
                                                                                                                                                                                                                                                      000530
                                                                                                                                                                                                                                                                  000540
                                                                                                                                                                                                                                                                                                                                                                            000630
 000320
                                    000350
                                                                                                                                                                                                                                                                                                              11S,19,7,16H MISSION TIME IS,F5.2,6H HOURS,7,11H DELTA T IS,F5.3//
                                                                                                                                                                                                                                                                                                  FORMAT(1H1,25HNUMBER OF TARGET TYPES IS,12,7,20H INITIAL LOADING
                                                                                                                                                                                                                                                                                                                                       RATE, 4X, 5HVALUE)
                                                                                                                                                                                                                               10**4
                                                                                                                                                                                                                                ×
                                                                                                                                                                                                                               7 I
                                                                                                                                                                                                                                +
                                                                                                                                                                                                                               * 10**5
                                                                                                                                                                                                                                                                                                                                      FORMAT (12H TARGET TYPE, 4X, 14HENCOUNTER
                                                                                                                                                                                                                             JPROD = I1 * 10**7 + I2 * 10**6+ I3
                                                                                                                                                                                                                                                                                                                                                 PRINT 306, (I, XLAM(I), V(I), I=1, M)
                                                                                                                                                                                                                                          + 17
                                                                                                                                                                                                                                                                                                                                                             FORMAT (17,10X,F10.5,6X,F5.2)
                                                                                                                                                                                                                                                                                       304, M, LOAD, T, DELTA
                                                                                                                                                                                                                                         * 10**2
                                  1,N1P
                                                                                                         1,N4P
                                                                                                                                1,N5P
                                                          1,N2P
                                                                                                                                                                                                                                                                            LOAD = ISUBS(IPROD)
                                                                                                                                                                              1000 NN7 = 1,N7P
                                                                                                                                                                                                     =1,N8P
                                                                                                                                                                                                                                        1+ I5 * 10**3 + I6
                                                                                                                                                       1000 NN6=1,N6P
                                                                                                                                                                                                                                                                 ISUBS(JJ) = JPROD
I=1, IPROD
                                                                                                                                  П
                                                                                                           H
                                                                                   H
                                    П
                                                           H
                                                                                                                                                                  = NN6 - 1
                                                                                                                                           = NN5 - 1
                                                                                                                                                                                                     1000 NN8
                                                         1000 NN2
                                                                                1000 NN3
                                                                                                        1000 NN4
                                                                                                                                1000 NN5
                                 1000 NN1
                                                                     = NN2 -
                                                                                                                                                                                                                                                      JJ = JJ + J
                                                                                            = NN3 -1
                                              = NN 1 -
           0.0
                                                                                                                   = NN4-1
                                                                                                                                                                                          = NN7-1
                                                                                                                                                                                                                 = NN8-1
                                                                                                                                                                                                                                                                                                                           PRINT 305
                                                                                                                                                                                                                                                                                                                                                                         RINT 837
                                                                                                                                                                                                                                                                                       PRINT
00 101
           ×(I) =
                       77=0
                                                         00
                                                                                                         00
                                                                                                                                                       00
                                                                                                                                                                                                     00
                                                                                 00
                                                                                                                     7 I
                                                                                                                                00
                                                                                                                                                                   9 I
                                                                                                                                          15
                                                                                                                                                                                                                                                                                                  304
                                                                                                                                                                                                                                                                1000
           101
                                                                                                                                                                                                                                                                                                                                      305
                                                                                                                                                                                                                                                                                                                                                             306
```

837	(1H1,40X,17HN K=1,M JN=1,LL	0064 0065 0066
836	836,K, T (12H	000
1	4=0.0	0000
(0 I=1, M	071
0	4=50ML = 1.0	007
	= T/DELTA	074
	×NDIV	0075
	2 I1=2,NDIV	076
	TIME =(I]-1)*DELTA	1
	S THE TIME F	0 7 8
	301,TME	079
301	T (//,17H	080
	3 I2=2, IP	081
	= ISUBS	082
	= ILOA	083
	5 I=1,8	084
	= ITEM/10**(8-	085
625	= ITEM - N(9
	= TEM	0087
	0 • 0=	000880
	<u>-</u>	0089
	,	600
	5 [4=],8	0091
ı	KFIKE(14013)=0 GIODII:VA INDIVATOS TUD NUMBER OF UPARONG OF EXOTE A FUSE	600
) ()	ENDED IN	0000930
		0095

000960 000970 000980 000990	0101	0103	0105	0108	0110	0112	0114	0116	0118	0121 0122 0123	0124 0125 0126 0127
= N(14) = (NL-1) = 106 15= 14 = 15-1 + 1S THE	7(JI4) 640,640,64 IRST = 0.0	0 TO 642 [RST = P(JI4,14,13) * V(13)	42 33083 = 1ECAD = 314×10 ××(8= DO 108 16 = 1,1PROD IF (JSUBS = 1SUBS(16)) 108,10	7 = 16 5 TO 110	ONTINUE ECOND=X(I7	Z=FIRST + IF (Z-A)1	11 A = Z KFIRE	06 CONT IF (ONTINUE ERM2 = TERM2 + Y (12)=TERM1 + TERM	T 302,(N(I AT (15H SU T 303, (I) AT (15H IF

	8H OF TYPE,12)	001280
103	103 CONTINUE	001290
	PRINT 320, (N(IY), IY=1,8), TIME, F(IPROD)	001300
320	320 FORMAT (//,1X,1HF,9I1,1H(,F5,3,3H) =,F10,5,/////)	001310
	DO 113 I=2, IPROD	001320
113	113 X(1)=F(1)	001330
102	102 CONTINUE	001340
	END	001350

~ ~ ~ ~

POLICY SAMPLE DUTPUT

10 TABLE

APPENDIX III

TABULATED VALUES OF fijs

BJS	RT	. ∓ •05	.10	.15	. 20	• 25
12345678901234567890		00445000000000000000000000000000000000	.09516 .09984 .10000 .10000 .10000 .10000 .10000 .10000 .10000 .10000 .10000 .10000 .10000 .10000	13929 14998 14998 15000 15000 15000 15000 15000 15000 15000 15000 15000 15000	18127 19879 19994 20000 20000 20000 20000 20000 20000 20000 20000 20000 20000 20000 20000 20000 20000	2120 21776 21778990000000000000000000000000000000000

BJS	RT = .30	•35	.40	•45	.50
12345678901234567890	25918 259672 299798 2299990 3300000 3300000 3300000 3300000 3300000 3300000 3300000 3300000 3300000	23333333333333333333333333333333333333	5291930 529191930 529191930 5291999900000000000000000000000000000000	311999000000000000000000000000000000000	3477 93666 93666 94999000000000000000000000000000000000

TABLE 11 VALUES OF f

```
        BJS
        RT = .55
        .60
        .65
        .70
        .75

        1
        .42305
        .45119
        .47795
        .50341
        .52763

        2
        .52878
        .57309
        .61658
        .65922
        .70099

        3
        .54724
        .59621
        .64492
        .69336
        .74150

        4
        .54997
        .59956
        .644937
        .69911
        .74879

        5
        .54997
        .59996
        .64993
        .69999
        .74985

        7
        .55000
        .60000
        .65000
        .70000
        .75000

        8
        .55000
        .60000
        .65000
        .70000
        .75000

        10
        .55000
        .60000
        .65000
        .70000
        .75000

        11
        .55000
        .60000
        .65000
        .70000
        .75000

        12
        .55000
        .60000
        .65000
        .70000
        .75000

        13
        .55000
        .60000
        .65000
        .70000
        .75000

        15
        .55000
        .60000
        .65000
        .70000
        .75000
```

BJS	RT = .80	.85	.90	.95
123456789011234567890112311567890	.55188 .7818338 .789338 .799990 .8000000 .8000000 .8000000 .8000000 .800000 .800000 .800000 .800000 .800000 .800000 .800000 .800000 .800000 .800000	57257 5875 5875 58875 588777 68877777 68877777 68877777 68877777 68877777 68877777 6887777777 68877777 68877777 68877777 68877777 68877777 68877777777 688777777 688777777 6887777777 6887777777 6887777777 6887777777 6887777777 6887777777 68877777777 68877777777 6887777777777	.59.43 .826.3826 .887.665 .889.9999 .899.0000 .900.0000 .900.0000 .900.0000 .900.000 .900.000 .900.000 .900.000 .900.000 .900.000 .900.0000 .900.000 .900.000 .900.000 .900.000 .900.000 .900.000 .900.0000 .900.000 .900.000 .900.000 .900.000 .900.000 .900.000 .900.0000 .900.000 .900.000 .900.000 .900.000 .900.000 .900.000 .900.0000 .900.000 .900.000 .900.000 .900.000000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000 .900.0000	.61321 .83115 .9346483 .9449990 .9449990 .95500000 .95500000 .95500000 .95500000 .95500000 .95500000 .95500000

# BJS	RT =	1		2	3	4	5
123456789 1011234 156789 1011234 1516789 20	.8 .9 .9 .9 .9 1.0 1.0 1.0 1.0 1.0 1.0	3212 9636 7666 9565 9931 9999 0000 0000 0000 0000 0000 0000	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	86466 78198 92486 97751 99408 99861 99971 99999 00000 00000 00000 00000 00000 00000 00000 00000	.95021 1.75106 2.32787 2.68064 2.86538 2.94930 2.98281 2.99471 2.99851 2.99962 2.99991 2.99998 3.00000 3.00000 3.00000 3.00000 3.00000 3.00000 3.00000 3.00000	.98168 1.89011 2.65200 3.21853 3.58970 3.80457 3.91524 3.96637 3.99587 3.99587 3.99587 3.99990 3.99997 3.99999 4.00000 4.00000 4.00000 4.00000 4.00000	.99326 1.95283 2.82818 3.56316 4.12266 4.50670 4.74452 4.87789 4.94598 4.97781 4.99151 4.99696 4.99898 4.99990 4.99997 4.99997 4.99997 5.00000 5.00000
BJS	'RT =	6	-	7	8	9	10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	1.99 3.74 5.46 5.55 5.55 5.99 5.99 5.99 5.99 5.99 5.9	9752 8017 1820 6700 8194 3699 8597 8597 8597 8597 89999 99999 99999	123455066666666666666666666666666666666666	.99909 .99179 .96216 .88039 .70740 .40669 .95698 .35827 .62918 .79868 .89720 .95055 .97755 .99036 .99608 .99848 .99944 .99980	.99966 1.99665 2.98289 3.94051 4.84088 5.64964 6.33627 6.88331 7.29076 7.57414 7.75825 7.87017 7.93397 7.96815 7.98541 7.99364 7.99736 7.99895 7.99986	.99988 1.99864 2.99241 3.97118 4.91622 5.80053 6.59375 7.26985 7.81420 8.22679 8.52080 8.71779 8.84202 8.91587 8.95734 8.97937 8.99048 8.99580 8.99580 8.99928	.99995 1.99946 2.99669 3.98635 4.95710 5.89001 6.75987 7.53965 8.20683 8.74890 9.16586 9.46908 9.67753 9.81306 9.89652 9.94526 9.97230 9.98658 9.99377 9.99722

BJS	RT = 11	12	13	14	15
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	.99998 1,99978 2,99857 3,99366 4,97855 5.94103 6.86242 7.71923 8.48724 9.14673 9.68684 10.10757 10.41888 10.63/59 10.78354 10.87615 10.93207 10.96426 10.98195 10.99124	.99999 1.99991 2.99939 3.99710 4.98950 5.96916 6.92334 7.83385 8.67880 9.43641 10.08918 10.62759 11.05162 11.37008 11.59806 11.75364 11.85493 11.91790 11.95532 11.97660	1.00000 1.9999/ 2.99974 3.99869 4.99495 5.98422 6.95833 7.90430 8.80455 9.63873 10.38/05 11.03369 11.57078 11.99774 12.32261 12.55900 12.72351 12.83304 12.9028/ 12.94554	1.00000 1.99999 2.99989 3.99942 4.99761 5.99208 6.97785 7.94623 8.88418 9.77478 10.59910 11.33906 11.98060 12.51615 12.94572 13.27636 13.52044 13.69324 13.81059 13.88710	1.00000 1.99999 2.99996 3.99974 4.99889 5.99610 6.98846 7.97046 8.93301 9.86316 10.74470 11.55994 12.29233 12.92912 13.46346 13.89537 14.23125 14.48239 14.66292 14.78770
BJS	RT = 16	17	18	19	20
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13 ABSTRACT

A pilot engaged in a strike mission against targets of opportunity is continually confronted with the decision as to how many and which of a savariable weapons should be expended against a variety of targets which are encountered. Associated with this decision problem is the problem of determining an optimal told for such a mission subject to the constraints of available payload and number of weapons stations on the aircraft. Certain assumptions are made concerning the distribution of targets within the target area which lead to a dynamic programming formulation of the decision problem. This yields a system of ordinary differential equations which are solvable recursively. In addition to the dynamic programming model, a sub-optimum determination of a "best load" is discussed. Although not as complete or precise as the dynamic programming method, this formulation and the aptable to squadron level decision making.

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